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Quantum confinement of magnetic-dipolar oscillations in ferrite discs

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Abstract
Because of confinement phenomena, semiconductor quantum dots show typical atomic properties such as discrete energy levels and shell structures. The energy eigenstates are described based on the Schrödinger-like equation for the electronic envelope wavefunctions. From the point of view of fundamental studies, the reduction of dimensionality in microwave ferrites brings into play new effects, which should be described based on the quantized picture and demonstrate, as a fact, the properties of artificial atomic structures. The intermediate position of magnetic-dipolar (or magnetostatic) oscillations in ferrite samples between 'pure' electromagnetic and spin-wave (exchange-interaction) processes reveals the very special behaviour of geometrical effects. In view of recent studies on the local-field effects for subwavelength systems, some aspects of magnetic-dipolar oscillations in a normally magnetized ferrite disc should be re-considered based on macroscopically quantized methods. The purpose of this paper is to develop macroscopically quantized phenomenological models for magnetostatic-wave ferrite discs based on the Schrödinger-like equation.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Magnetostatic (MS) mode characterization in ferrite samples appears to be a relatively straightforward and old problem in magnetism. Some aspects, however, show very interesting features of such oscillations, which demand macroscopically quantized methods for clearer understanding and new implementation. Among these aspects there are the atomic-like spectral characteristics in normally magnetized thin-film ferrite discs. In the last few years, there has been renewed interest in high frequency dynamic properties of finite size magnetic structures. In a series of new publications, confinement phenomena of high-frequency magnetization dynamics in magnetic particles have been the subject of much experimental and theoretical attention (see [1] and references therein). In particular, these works are devoted to the important study of localized spin-wave spectra, but do not focus on the energy eigenstates of a whole ferrite-particle system. The localization of spin-wave modes considered in these papers do not provide any mechanism of macroscopic quantization similar to semiconductor quantum wells. Until now there have been no (to the best of our knowledge) phenomenological models of a ferrite particle with high-frequency magnetization dynamics which use the effective-mass approximation and the Schrödinger-like equation to analyse the energy eigenstates of a whole ferrite-particle system.

Multiresonance magnetic oscillations in small ferrite spheres excited by external RF magnetic fields were experimentally observed for the first time by White and Solt in 1956 [2]. Subsequently, experiments with other forms of ferrite specimen were carried out. Without intending to survey such experiments, we would like to call the reader's attention to the fundamental difference between the experimental absorption spectra of the sphere-form [2] and the disc-form ferrite resonators [3, 4]. The δ-functional character of the multi-resonance spectra one can see in the case of a ferrite disc resonator leads to a clear conclusion that the energy of a source of a DC bias magnetic field is absorbing 'by portions' or, in other words, discretely. By contrast, the spectrum of a ferrite sphere is characterized by very few and very 'spreading' absorption peaks. The effect of conversion of electromagnetic power into MS-wave power spectra (with differences of the order of 2–4 in the wavelength) in non-spherical ferrite samples has been the topic of serious experimental and theoretical investigations [4–6]. The main aim of these works was to show that the multi-resonance MS oscillations could chiefly be observed due to the non-uniform internal DC magnetic field in disc-shaped resonators. Recently, we have shown that MS oscillations in a small ferrite disc resonator can be characterized by a discrete spectrum of energy eigenstates [7]. This fact allows the analysis of the MS oscillations in a similar way to quantum mechanical problems. It gives a basis for a clearer understanding of the nature of the observed multi-resonance spectrum. From this standpoint, the role of the non-uniform DC magnetic field should be considered as an additional factor that, certainly, can lead to distortion of an initial discrete spectrum in a ferrite disc but does not imply a fundamental character.

Ferrite MS-wave samples are well localized in space, their extension is assumed to be much smaller than the variation length of the electromagnetic field. In fact, experiments [2–4] demonstrate the effects of interaction of microwave-cavity electromagnetic fields with a point-like object. So a MS-wave ferrite resonator interacts with a local, or quasistatic, cavity field. Quasistatics means that the characteristic specimen size l is much less than the free-space electromagnetic wavelength $\lambda \sim c/\omega$ [8]. The main origin of the problem (which we are faced with in our attempts to give a proper explanation of the multiresonance experimental spectra) can be attributed to the crucial role played by the *evanescent components* of the field (the MS field) in the region close to the sample. This is the problem related to non-radiative energy transfers in the quasistatic region of the cavity field. In complete analogy with the tunnel effect for electrons, these evanescent components can lead to magnetic tunnel effects. In the quasistatic range (of the cavity RF field), the accurate treatment of evanescent waves requires one to deal carefully with the boundary conditions and to include appropriate magnetic frequency responses. One has effects involving a high density of evanescent waves existing in the quasistatic region of the cavity field.

The optical local-field response of optical microresonators and semiconductor quantum dots is now an elaborated subject (see [9] and references therein). The optical susceptibility of semiconductor quantum dots is found from the light–matter coupling Hamiltonian and quantum confinement effects splitting the bulk material properties into a series of discrete energy levels. MS oscillations in ferrite particles exist due to the essential temporal dispersion of magnetic susceptibility [8]. Similar to semiconductor quantum dots showing an optical local-field response, point-like ferrite particles are resonant scatterers showing the cavity local-field response.

The statement that confinement phenomena for MS oscillations in a normally magnetized ferrite disc demonstrate typical atomic-like properties of discrete energy levels could be illustrated by an analysis of experimental absorption spectra. The main feature of multi-resonance line spectra in [3, 4] is the fact that high-order peaks correspond to lower quantities of the bias DC magnetic field. Physically, the situation looks as follows. Let $H_0^{(A)}$ and $H_0^{(B)}$ be, respectively, the upper and lower values of a bias magnetic field corresponding to the borders of a region. We can estimate the total depth of a 'potential well' as: $\Delta U = 4\pi M_0(H_0^{(A)} - H_0^{(B)})$, where M_0 is the saturation magnetization. Let $H_0^{(1)}$ be a bias magnetic field corresponding to the *main absorption peak* in the experimental spectrum ($H_0^{(B)} < H_0^{(1)} < H_0^{(A)}$). When we put a ferrite sample into this field, we supply it with the energy $4\pi M_0 H_0^{(1)}$. To some extent, this is a pumping-up energy. *Starting from this level*, we can excite the entire spectrum from the main mode to the high-order modes. As the value of the bias magnetic field decreases, the 'particle' obtains the *higher levels of negative energy*. One can estimate the negative energies necessary for transitions from the main level to upper levels. For example, to have a transition from the first level $H_0^{(1)}$ to the second level $H_0^{(2)}$ ($H_0^{(B)} < H_0^{(2)} < H_0^{(1)} < H_0^{(A)}$) we need the density energy surplus $\Delta U_{12} = 4\pi M_0(H_0^{(1)} - H_0^{(2)})$. The situation resembles increasing the negative energy of the hole in semiconductors when it 'moves' from the top of a valence band. In classical theory, negative-energy solutions are rejected because they cannot be reached by a continuous loss of energy. But in quantum theory, a system can jump from one energy level to a discretely lower one, so the negative-energy solutions cannot be rejected out of hand. When one continuously varies the quantity of the DC field H_0, for a given quantity of frequency ω, one sees a discrete set of absorption peaks. This means that one has discrete-set levels of potential energy. The line spectra appear due to the quantum-like transitions between energy levels of a ferrite disc-form particle. As a quantitative characteristic of permitted quantum transitions there is the probability, which defines the intensities of the spectral lines. The quantized-like transitions for MS oscillations in a normally magnetized thin-film ferrite disc were demonstrated in recent experimental studies [10].

In [11] we showed that because of the discrete energy eigenstates of MS oscillations resulting from structural confinement in a ferrite disc, one can describe the oscillating system as a collective motion of quasiparticles—the light magnons. The light magnon distribution is defined as the probability density distribution function. The confined phenomena of the light magnon oscillations in normally magnetized thin-film ferrite discs demonstrate very specific properties of artificial atomic structures. One of these specific properties is the eigen electric moments (the anapole moments) accompanying MS oscillations. Such eigen-electric-moment oscillations were predicted in [12] and experimentally verified in [10]. Another aspect of the light-magnon motion character is the microwave magnetoelectric (ME) effect observed in recent experiments [13].

In this paper we develop macroscopically quantized phenomenological models for MS oscillations in a ferrite disc resonator. In view of the correct formulation of the energy orthogonality relations, we analyse the problem of boundary conditions and mutual matching between the frequency and magnetic-field spectra. We discuss the question of possible quantization of magnetization for MS oscillations. The final part of the paper is devoted to the MS-mode spectral problem in a ferrite disc with a non-homogeneous DC magnetic field.

2. Resonance frequencies of a ferrite disc resonator

A model of a normally magnetized open ferrite resonator is shown in figure 1. This is a ferrite disc without any perfect electric or perfect magnetic walls. Since the disc has a small

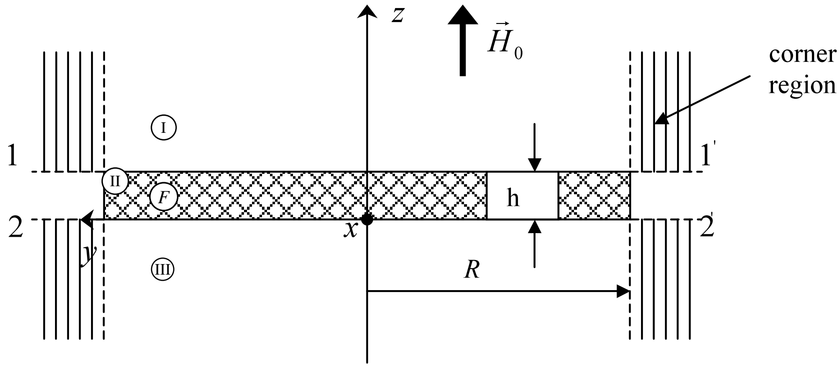

Figure 1. A normally magnetized open ferrite disc resonator.

thickness/diameter ratio, separation of variables is possible. In the case of such an assumption, we exclude, in fact, the influence of the edge regions.

In a ferrite-disc resonator with a small thickness to diameter ratio, the monochromatic MS-wave potential function ψ is represented as [7]:

$$\psi = \sum_{p,q} A_{pq}\tilde{\xi}_{pq}(z)\tilde{\varphi}_q(\rho, \alpha), \tag{1}$$

where A_{pq} is the MS mode amplitude, $\tilde{\xi}_{pq}(z)$ and $\tilde{\varphi}_q(\rho, \alpha)$ are dimensionless functions describing, respectively, the 'thickness' (z coordinate) and 'in-plane', or 'flat' (radial ρ and azimuth α coordinates), MS modes. For a certain type of 'thickness' mode (in other words, for a given quantity p), every 'flat' mode is characterized by its own function $\tilde{\xi}_q(z)$.

Because of separation of variables, one can impose independently the electrodynamical boundary conditions—the continuity conditions for the MS potential ψ and for the normal components of the magnetic flux density—on a lateral cylindrical surface ($\rho = R, 0 \leqslant z \leqslant h$) and plane surfaces ($\rho \leqslant R, z = 0, z = h$). As a result, we have to solve a system of the following two equations:

$$\tan(\beta^{(F)}h) = -\frac{2\sqrt{-\mu}}{1+\mu} \tag{2}$$

and

$$(-\mu)^{\frac{1}{2}}\frac{J'_\nu}{J_\nu} + \frac{K'_\nu}{K_\nu} - \frac{\mu_a \nu}{|\beta^{(F)}|R} = 0. \tag{3}$$

Here μ and μ_a are, respectively, the diagonal and off-diagonal components of the permeability tensor, $\beta^{(F)}$ is the wavenumber of a MS wave propagating in a ferrite along the bias magnetic field, J_ν, J'_ν, K_ν and K'_ν are the values of the Bessel functions of order ν and their derivatives (with respect to the argument) on a lateral cylindrical surface ($\rho = R, 0 \leqslant z \leqslant h$).

Equations (2) and (3) correspond, respectively, to characteristic equations for MS waves in a normally magnetized ferrite slab [14] and in an axially magnetized ferrite rod [15]. To obtain eigenfrequencies of a ferrite disc resonator one has to solve a system of two equations, equations (2) and (3), for given values of h, R and ν. The solutions for oscillating MS modes take place only for $\mu < 0$. This means that the admissible frequency region is restricted by frequencies ω_1 and ω_2 ($\omega_1 \leqslant \omega \leqslant \omega_2$), where $\omega_1 = \gamma H_i$ and $\omega_2 = \gamma[H_i(H_i + 4\pi M_0)]^{1/2}$. Here γ is the gyromagnetic ratio and H_i is the internal DC magnetic field. It becomes clear (see equation (3)) that one should have *different resonances for the left-hand and the right-hand circularly polarized oscillations* (having different signs of ν).

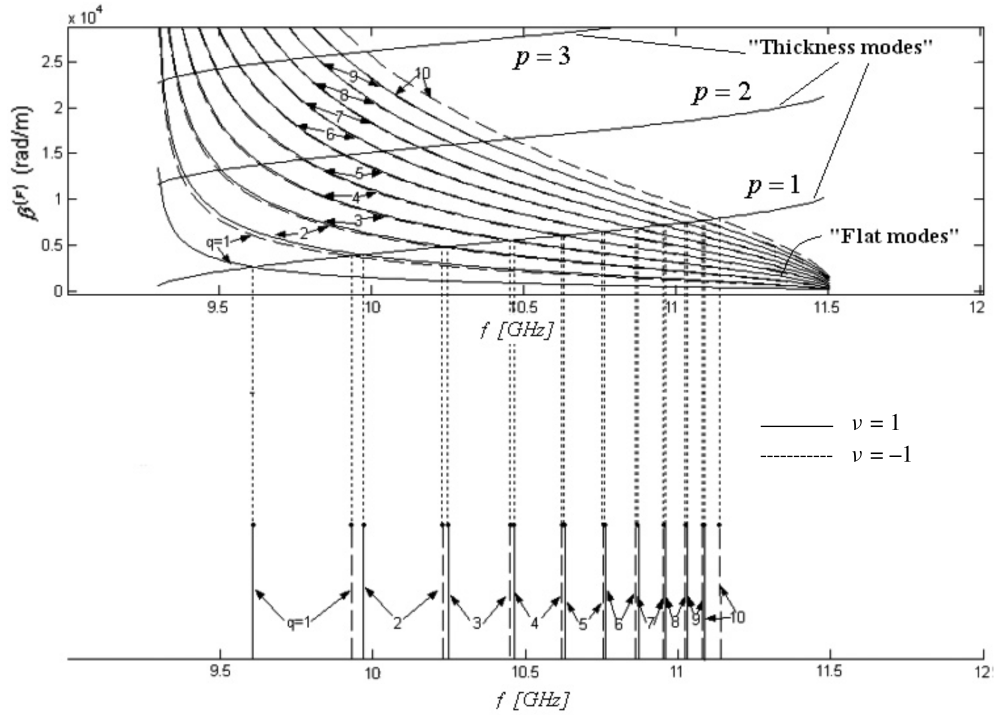

Figure 2. Resonance frequencies of a ferrite disc resonator. (a) The graphical solutions of equations (2) and (3). (b) The spectrum of resonance peaks for a fundamental 'thickness mode'.

Figure 2(a) illustrates the graphical solutions of equations (2) and (3) obtained for a set of 'thickness modes' (p numbers) and different 'in-plane (flat) modes' with $\nu = \pm 1$ and with a number of radial variations (q numbers). An analysis was made using the disc data given in [4]: $4\pi M_0 = 1792$ G, $2R = 3.98$ mm, $h = 0.284$ mm. Calculations were made for the bias DC magnetic field $H_0 = 5.02$ kOe. One can see that in our case of a ferrite disc with a small thickness/diameter ratio, the spectrum of 'thickness modes' is very 'rare' compared to the 'dense' spectrum of 'flat modes'. The entire spectrum of 'flat modes' is completely included in the wavenumber region of a fundamental 'thickness mode'. This means that the spectral properties of a resonator can be entirely described based on consideration of only a fundamental 'thickness mode'. The spectrum of resonance peaks corresponding to solutions of equations (2) and (3) for a fundamental 'thickness mode' is shown in figure 2(b) by vertical lines. There is clear evidence for a strong difference in positions of peaks with positive and negative signs of ν. Such a difference of resonances for the left-hand and the right-hand circularly polarized oscillations reveals a problem for MS-mode orthogonality relations and difficulties in the analysis of the energy spectra.

3. Energy spectra of a ferrite disc resonator

MS oscillations in a one-dimensional linear structure are completely described by scalar wavefunction ψ. In the case of a lossless structure, one has the following differential equation [11]:

$$a^{(1)}(z)\frac{\partial^2\psi(z,t)}{\partial z^2}+a^{(2)}(z)\psi(z,t)=\frac{\partial\psi(z,t)}{\partial t}. \tag{4}$$

This is the Schrödinger-like equation. One can consider equation (4) as the *stationary-state* equation.

An axially magnetized ferrite cylinder is a quasi one-dimensional linear structure. For parameters not dependent on a longitudinal coordinate, MS oscillations are described by equation (4) with coefficients $a^{(1)}$ and $a^{(2)}$ independent of z. The feature of such a waveguide structure is the fact that there are two cutoff frequencies ω_1, ω_2. For a given frequency in the frequency region between the cutoff frequencies ($\omega_1 \leqslant \omega \leqslant \omega_2$), one has a *discrete spectrum of propagating MS modes* [7, 15]. For a monochromatic process ($\psi \sim e^{i(\omega t-\beta z)}$) we have an infinite set of differential equations (that are all similar to equation (4)) written for waveguide modes. For the nth waveguide mode we obtain from equation (4) for frequency ω:

$$-a_n^{(1)}\beta_n^2+a_n^{(2)}=i\omega. \tag{5}$$

For harmonic processes, coefficients $a^{(1)}$ and $a^{(2)}$ should be imaginary quantities. There is, however, certain vagueness as to how to determine these coefficients.

Let us represent a MS-potential function as a quasi-monochromatic quantity:

$$\psi=\psi^{(\max)}(z,t)e^{i(\omega t-\beta z)}, \tag{6}$$

where the amplitude $\psi^{(\max)}(z,t)$ is a smooth function of the longitudinal coordinate and time. For the quasi-monochromatic MS-wave process of mode n, the average energy of the MS waveguide section, restricted by $z = z_1, z_2$, is characterized as [7, 11]

$$\bar{W}_n=-\frac{1}{4a_n^{(1)}}i\omega\mu_0\int_{z_1}^{z_2}\int_S\psi_n\psi_n^*\,ds\,dz+C, \tag{7}$$

where C is an arbitrary quantity not dependent on time. It is possible to normalize the process in a supposition that constant C is equal to zero. One can see that coefficient $a_n^{(2)}$ is not included in the expression of average energy. The only coefficient included in this expression is coefficient $a_n^{(1)}$. Another important conclusion following from equation (7) is that for any coefficient $a_n^{(1)}$ the energy can be normalized with respect to the known ψ_n eigenfunctions.

The fact that coefficient $a^{(2)}$ is not included in the expression of average energy gives us the possibility to consider different cases based on certain physical models. One can see that when $a^{(2)} \equiv 0$, equation (4) resembles the Schrödinger equation for 'free particles'. For the case of $a_n^{(2)} \equiv 0$, coefficients $a_n^{(1)}$ are found as (see equation (5)):

$$a_n^{(1)}=-\frac{i\omega}{\beta_n^2}. \tag{8}$$

We define a notion of the *normalized average MS energy of mode n* as the average (on the RF period) energy of the MS waveguide section with unit length and unit characteristic cross section. This energy for a mode with unit amplitude is expressed based on equations (7) and (8) as:

$$E_n^{(\mathrm{lm})}=\tfrac{1}{4}g\mu_0\beta_n^2, \tag{9}$$

where g is the unit dimensional coefficient. The superscript (lm) used in equation (9) means the 'light magnon' [7, 11].

The MS-potential wavefunctions show the possible eigenstates of a system. It follows that the energy quantization (described by the MS-potential properties) is regarded to a collective effect of quasiparticles. When dealing with quasiparticles, it is standard to introduce the concept of an 'effective mass', i.e. a quantity with dimension of mass, characterizing dynamic properties of a quasiparticle. A quasiparticle may behave differently in different conditions, so that 'effective masses' proliferate. The process of MS-wave propagation is considered as the motion process of certain quantum quasiparticles having quantization of energy and characterized by certain *effective masses*. We call these quasiparticles the light magnons. The meaning of this term arises from the fact that the effective masses of the light magnons should be much less than the effective masses of the (real, 'heavy') magnons—the quasiparticles existing due to the exchange interaction. The states of the light magnons are described based on the so-called *translational eigenfunctions*. For these translational eigenfunctions energy is proportional to a squared wavenumber [16]. When we juxtapose equation (4) with the Schrödinger equation for 'free particles' ($a^{(2)} \equiv 0$), we get the following expression for an effective mass of a light magnon:

$$\left(m_{\mathrm{eff}}^{(\mathrm{lm})}\right)_n = \frac{\hbar}{2}\frac{\beta_n^2}{\omega}. \tag{10}$$

This expression looks very similar to an effective mass of the (real, 'heavy') magnon for spin waves with the quadratic character of dispersion [17].

In an infinite-ferrite-rod MS-wave waveguide for given frequency ω' ($\omega_1 \leqslant \omega' \leqslant \omega_2$), one has a flow of quasiparticles with different effective masses and different kinetic energies. For another frequency $\omega'' \neq \omega'$ ($\omega_1 \leqslant \omega'' \leqslant \omega_2$) we have a flow of other quasi-particles differing from previous ones by effective mass and kinetic energy. At a certain frequency, the total energy of non-interacting quasiparticles is equal to a sum of energies of separate quasiparticles:

$$E_{\mathrm{tot}}^{(\mathrm{lm})} = \sum_n E_n^{(\mathrm{lm})}. \tag{11}$$

Since the spectrum of 'thickness modes' is very 'rare' compared to the 'dense' spectrum of 'flat modes', the spectral properties of a ferrite disc resonator with a small thickness/diameter ratio can be entirely described based on consideration of only a fundamental 'thickness mode'. To find the average energy of the MS mode q in a ferrite disc, one can extend equation (7) as follows:

$$\bar{W}_q = -\frac{1}{4}\mathrm{i}\omega\mu_0 \int_S \left[\frac{1}{(a_q^{(1)})^{(\mathrm{D})}} \int_{-\infty}^{0} \psi_q \psi_q^* \,\mathrm{d}z + \frac{1}{(a_q^{(1)})^{(\mathrm{F})}} \int_0^h \psi_q \psi_q^* \,\mathrm{d}z + \frac{1}{(a_q^{(1)})^{(\mathrm{D})}} \int_h^{\infty} \psi_q \psi_q^* \,\mathrm{d}z \right] \mathrm{d}s + C, \tag{12}$$

where superscripts (D) and (F) mean, respectively, the dielectric ($-\infty \leqslant z \leqslant 0$; $h \leqslant z \leqslant \infty$) and ferrite ($0 \leqslant z \leqslant h$) regions. It becomes clear that only the term corresponding to the ferrite region (where we have propagating MS waves) gives a real quantity. For a given quantity H_0, in the case of an infinite ferrite rod, we have a set of transitionally moving light magnons. For a given quantity H_0, in the case of a ferrite disc resonator, there is a set of light magnons having *reflexively-translational* motion between the planes $z = 0$ and h. Since at a certain frequency there are two waves propagating forward and back with respect to the z-axis, the average energy will be twice that of the energy expressed by equation (9). One has the following expression for the light-magnon average energy of 'flat' mode q in a normally magnetized ferrite disc:

$$E_q^{(\mathrm{lm})} = \tfrac{1}{2} g\mu_0 (\beta_q^{(\mathrm{F})})^2, \tag{13}$$

where $\beta_q^{(\mathrm{F})}$ is a MS-wave propagation constant in a ferrite of mode q.

With consideration of the MS-wave process as the motion of quasiparticles and the definition of effective masses of these quasiparticles, one cannot, however, calculate the energy spectra because of an ambiguity arising from differences in positions of peaks with positive and negative signs of ν. Such a difference, shown in figure 2, reveals a contradiction in the formulation of the orthonormality relations. In the analysis, it should be supposed that function $\tilde{\varphi}$ is a single valued function for angle α varying from 0 to 2π. So, before starting computations of these energy levels one should overcome the difficulties related to the difference in positions of peaks with positive and negative signs of ν, mentioned above in section 2.

The homogeneous electrodynamics boundary conditions at $\rho = R$ demand continuity for $\tilde{\varphi}$ and continuity for the radial component of the magnetic flux density. The last boundary condition is described as:

$$\mu\left(\frac{\partial\tilde{\varphi}}{\partial\rho}\right)_{\rho=R^-} - \left(\frac{\partial\tilde{\varphi}}{\partial\rho}\right)_{\rho=R^+} = -\frac{\mu_a}{R}\nu(\tilde{\varphi})_{\rho=R^-}. \tag{14}$$

This is a special boundary condition on the border 'in-plane' contour L. Really, the 'flat' functions $\tilde{\varphi}$ determined by the Bessel functions should be degenerated with respect to a sign of ν. At the same time, in accordance with the first-order differential equation (14), the functions $\tilde{\varphi}$ are dependent on the sign of ν. Because of the boundary conditions functions $\tilde{\varphi}$ cannot be considered as *single-valued functions*. At the same time, following axioms of quantum mechanics [16], each state function, as well as a superposition of the state functions *must be a single-valued analytic expression* satisfying the boundary conditions for the given system. The fact that solution of our problem is dependent on both a modulus and a sign of ν raises a question about the validity of the energy orthonormality relation for functions $\tilde{\varphi}$.

The energy orthonormality relations can be obtained based on the so-called *essential boundary conditions* used in variational methods. Since a two-dimensional ('in-plane') differential operator $\hat{G}_\perp$ [11] contains $\nabla_\perp^2$ (the two-dimensional, 'in-plane', Laplace operator), a double integration by parts (the Green theorem) on S—a square of an 'in-plane' cross section of an open ferrite disc—of the integral $\int(\hat{G}_\perp\tilde{\varphi})\tilde{\varphi}^*\,\mathrm{d}S$, gives the following boundary condition:

$$\mu\left(\frac{\partial\tilde{\varphi}}{\partial\rho}\right)_{\rho=R^-} - \left(\frac{\partial\tilde{\varphi}}{\partial\rho}\right)_{\rho=R^+} = 0. \tag{15}$$

For operator $\hat{G}_\perp$, the boundary condition of the MS-potential continuity together with boundary condition (15) are the essential boundary conditions [18]. When the essential boundary conditions are used, the MS-potential eigenfunctions of operator $\hat{G}_\perp$ form a *complete basis in an energy functional space*, and the functional describing an average quantity of energy has a minimum at the energy eigenfunctions [18]. The boundary conditions of the MS-potential continuity together with boundary condition (14) are the so-called *natural* boundary conditions [18]. The possibility of using the functions from the complete energy space when inhomogeneous boundary conditions take place is discussed in many variational problems [18, 19]. The main feature of natural boundary condition (14) arises from the quantity of an azimuth magnetic field. One can see that this is a *singular* field, which exists only in an infinitesimally narrow cylindrical layer abutting (from the ferrite side) to the ferrite–dielectric border. An azimuth magnetic field defines a surface magnetic current on the ferrite–dielectric border. This current, being described by the *double-valued* functions, gives *anapole moments* for MS oscillations in a ferrite disc [12].

We will calculate now the resonance peak positions in a ferrite disc resonator based on the essential boundary conditions considered above. In this case of boundary conditions one does not have the difference of resonances for the left-hand and the right-hand circularly polarized oscillations. The resonance peak positions should be obtained based on the graphical solution of equation (2) and the modified form of equation (3). The latter, taking into account the essential boundary conditions, is represented as

$$(-\mu)^{\frac{1}{2}}\frac{J'_\nu}{J_\nu} + \frac{K'_\nu}{K_\nu} = 0. \tag{16}$$

The feature of this equation is the fact that at the point where $\mu = -1$, one has an identity.

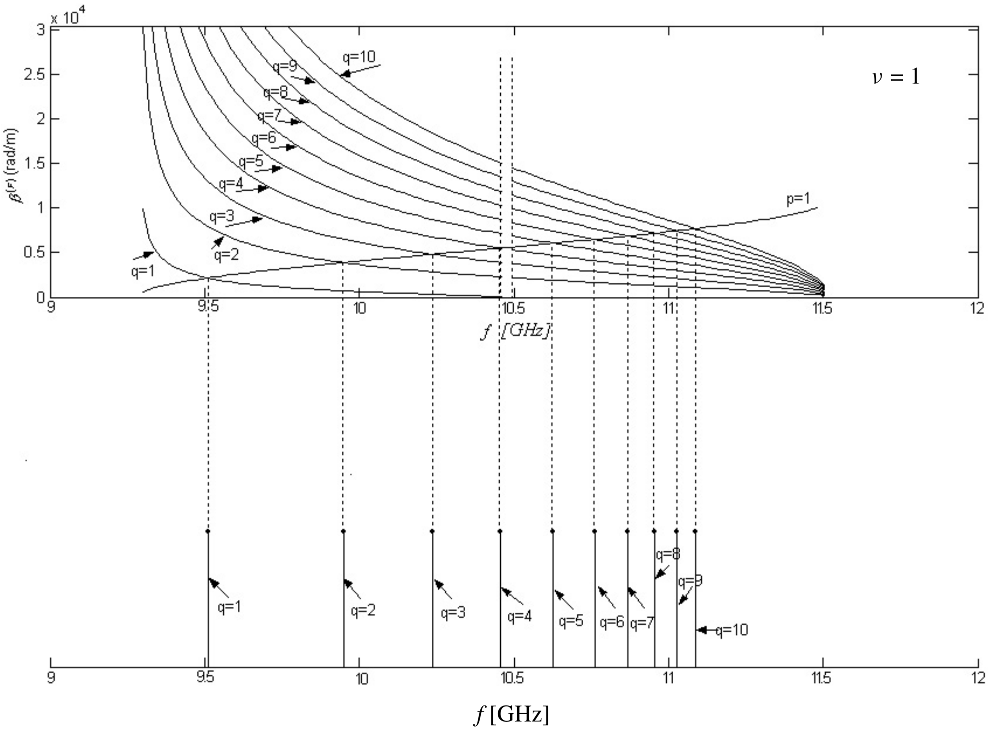

Figure 3. Resonance spectrum of a ferrite disc with respect to frequency (the essential boundary conditions). (a) The graphical solutions of equations (2) and (16). (b) The spectrum of resonance peaks for a fundamental 'thickness mode'.

Figure 3(a) illustrates the graphical solutions of equations (2) and (16) obtained for the main 'thickness mode' and different 'in-plane (flat) modes' calculated for Bessel functions of order $\nu = 1$ and with a number of radial variations (q numbers). An analysis was made using the same data as for calculations shown in figure 2. At the frequency corresponding to the quantity $\mu = -1$ one has a break. The spectrum of resonance peaks corresponding to solutions of equations (2) and (16) for a fundamental 'thickness mode' is shown in figure 3(b) by vertical lines.

Usually in experiments the spectral properties of small ferrite resonators are exhibited with respect to a quantity of bias magnetic field, remaining a quantity of frequency without any variations. The graphical solutions of equations (2) and (16) obtained for the same data of disc parameters as calculations in figure 3, but with respect to the DC magnetic field H_0, are shown in figure 4 for different quantities ν. The working frequency is $\frac{\omega}{2\pi} = 9.51$ GHz. The break takes place at the magnetic field where $\mu = -1$.

For known quantities $\beta_q^{(F)}$ (see figure 4), the energy levels can be calculated based on equation (13). The positions of quantities $E_q^{(lm)}$—the normalized energies of the light-magnon collection—corresponding to different 'flat modes' were calculated and shown in [11]. The modes are characterized by numbers q and quantities ν. The classification is the following. For $\nu = 1$, one has the energy levels for the dipole-type light magnon collection, for $\nu = 2$, the quadrupole-type light magnon collection, and for $\nu = 3$ there is the hexapole-type light magnon collection. Also in [11], effective masses of different-type light magnons, calculated based on equation (10), are shown. MS-potential distributions with respect to axial, radial and

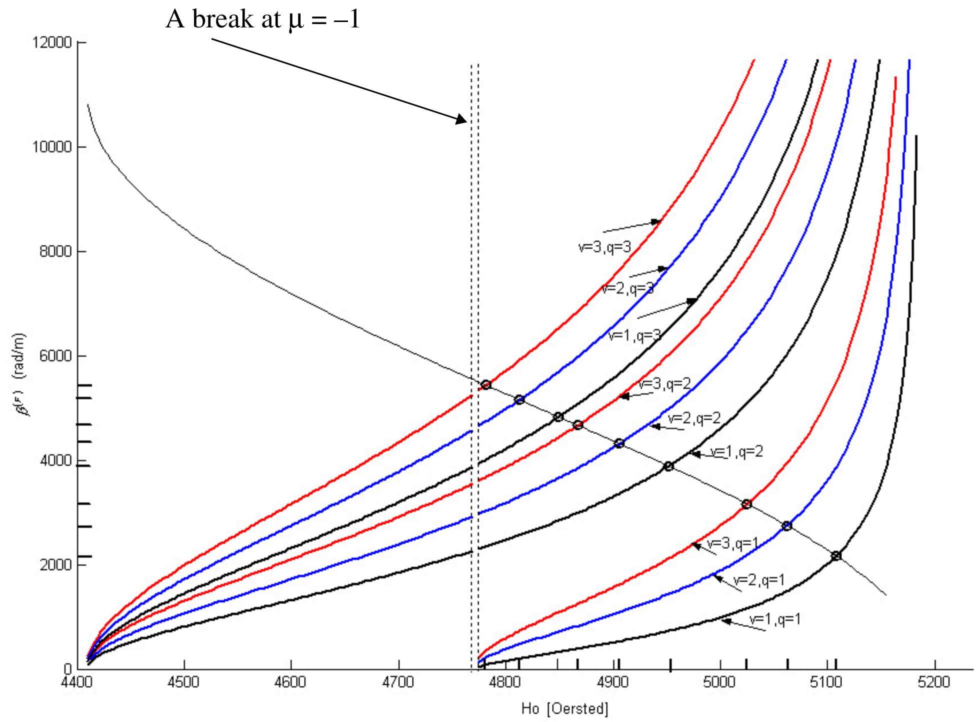

Figure 4. The graphical solutions of equations (2) and (16) with respect to the DC magnetic field for the main 'thickness' and different 'in-plane' MS modes.

azimuth coordinates shown, respectively, in figures 5–7 of the paper illustrate the behaviour of the light magnons in a ferrite disc resonator. In MS-wave processes, the MS-potential function can be considered as the probability distribution function [7]. The probability density distribution function $\tilde{\varphi}\tilde{\varphi}^*$ shows the light magnon distribution.

The energies found from equation (13) can be considered as 'kinetic energies'. At the same time, the fact that the spectral properties are exhibited with respect to quantities of a bias magnetic field means variations of 'potential energy' of a ferrite sample. When one continuously varies the quantity of the DC field H_0, for a given quantity of ω, one sees a discrete set of absorption peaks. This means that one has the discrete-set levels of potential energy. It is a very crucial fact that the jumps between the potential levels are controlled (are governed) by the discrete transitions between the quantum states of the light magnons. For known quantities $H_0^{(\mathrm{A})}$ and $H_0^{(\mathrm{B})}$, corresponding to the upper and lower values of a bias magnetic field region where $\mu < 0$, we calculate the total depth of a 'potential well'. For working frequency $\frac{\omega}{2\pi} = 9.51$ GHz and saturation magnetization $4\pi M_0 = 1792$ G—the data of the Yukawa and Abe's experiments [4]—we have:

$$\Delta U = 4\pi M_0\left(H_0^{(\mathrm{A})} - H_0^{(\mathrm{B})}\right)$$
$$= 780\ \mathrm{Oe} \times 1792\ \mathrm{G} = 1.4 \times 10^6\ \mathrm{ergs\ cm^{-3}} = 14 \times 10^4\ \mathrm{J\ m^{-3}}. \qquad (17)$$

The first three levels ($q = 1, 2, 3$) of negative potential energy, calculated as $U = 4\pi M_0(H_0|_{q=1,2,3} - H_0^{(\mathrm{A})})$, where the quantities $H_0|_{q=1,2,3}$ are found from the first-three-peak positions in the magnetic-field spectra (see figure 4), are shown in figure 8 for $\nu = 1$. For every level of potential energy, the corresponding quantities of the light-magnon normalized

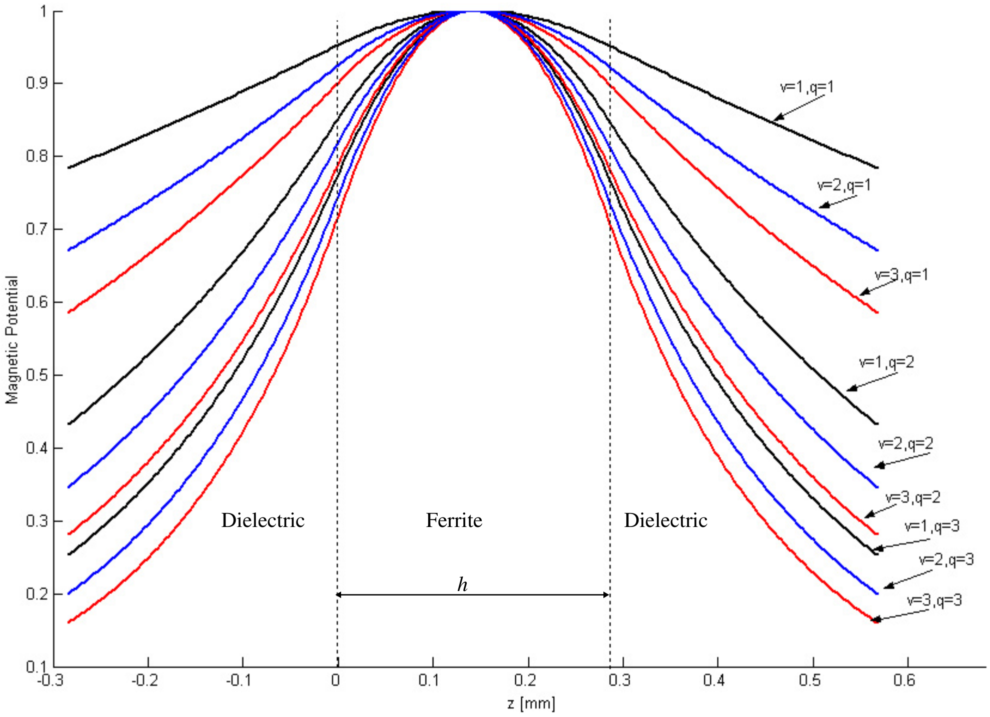

Figure 5. MS-potential distribution with respect to an axial coordinate.

energies are pointed out. The quantized quantities of the light magnon kinetic energy take place only on certain discrete levels of potential energy. Based on the above analysis, we can find certain appropriateness between the light-magnon kinetic energy levels and the potential energy levels. Such appropriateness between the $E_q^{(\mathrm{lm})}$ and $|U_q|$ levels is illustrated in figure 9 for different types of light magnon.

4. Correlation between the frequency and magnetic-field spectra in a ferrite disc resonator

In a general consideration, the physical justification for definition of the energy levels in a ferrite disc should be based on the notion of the *density matrix* used in quantum mechanics [16, 20]. As we discussed above, in an infinite ferrite rod the ψ function can be expanded by membrane functions $\tilde{\varphi}$ of MS-wave waveguide modes for a monochromatic process ($\psi \sim \mathrm{e}^{\mathrm{i}\omega t}$). In this case we have an infinite set of differential equations (that are all similar to equation (4)) written for waveguide modes. This is not the situation one may see in a ferrite disc resonator.

Let us consider the case of a constant-value bias magnetic field H_0. Every resonance mode in figure 3 is described by equation (4). However, since every resonance peak is characterized by its own frequency, one can suppose that there are no complete-set membrane functions $\tilde{\varphi}$ of MS modes. For a given quantity of bias magnetic field H_0, let us introduce the following function $\Theta(\omega_a, \omega_b)$ written for the 'in-plane' functions $\tilde{\varphi}$:

$$\Theta(\omega_a, \omega_b) = \int_S \tilde{\varphi}^*(\omega_a)\tilde{\varphi}(\omega_b)\,\mathrm{d}s, \tag{18}$$

where $\omega_{a,b}$ are frequencies of some two resonance peaks (the peaks numbered as a and b)

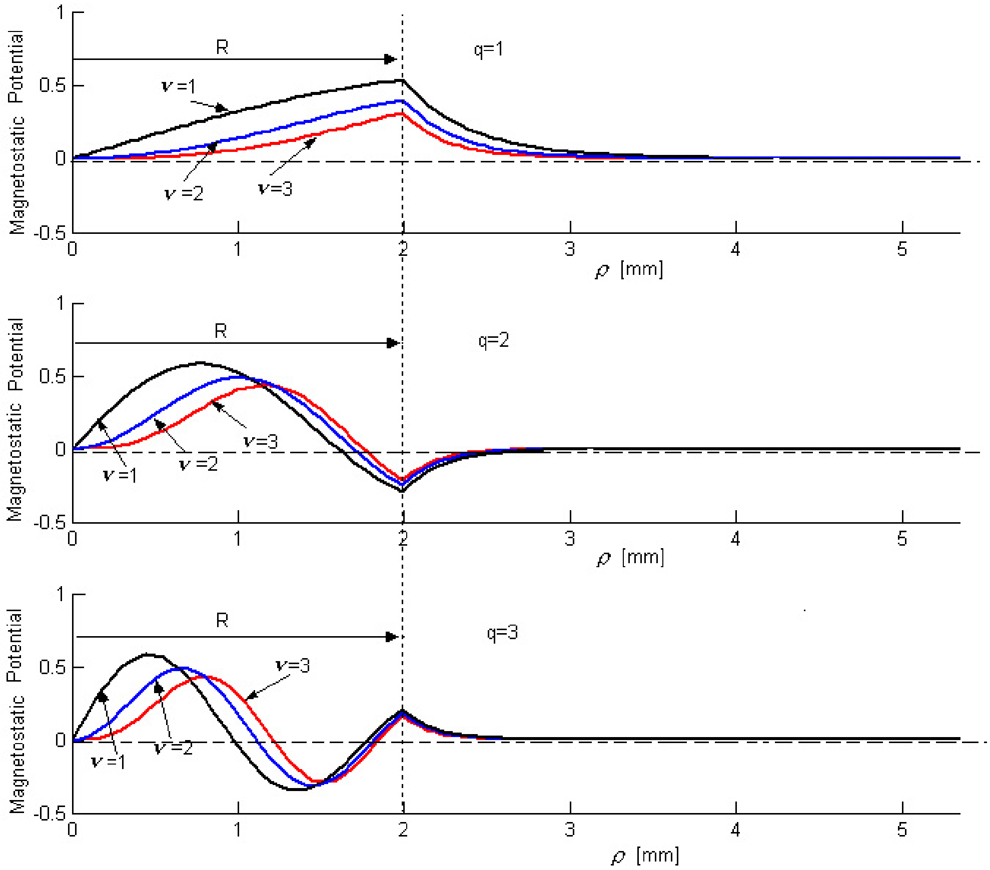

Figure 6. MS-potential distributions with respect to a radial coordinate.

in figure 3. By analogy with the quantum mechanics problems, we will call function $\Theta(\omega_a, \omega_b)$ the density matrix. Evidently the density matrix is characterized by the Hermitian property:

$$\Theta^*(\omega_a, \omega_b) = \Theta(\omega_a, \omega_b). \tag{19}$$

Diagonal elements of the density matrix are defined as:

$$\Theta(\omega_a, \omega_a) = \int_S |\tilde{\varphi}(\omega_a)|^2 \, \mathrm{d}s. \tag{20}$$

In accordance with figure 3 one can see that the number of a resonance peak corresponds to the number of an 'in-plane' function. So instead of equation (18) one should write:

$$\Theta_{m,n}(\omega_m, \omega_n) = \int_S \tilde{\varphi}_m^*(\omega_m)\tilde{\varphi}_n(\omega_n) \, \mathrm{d}s. \tag{21}$$

There is no foundation for stating *a priori* that, for a given quantity of bias magnetic field H_0, when frequencies ω_m and ω_n are different (see figure 3), functions $\tilde{\varphi}_m$ and $\tilde{\varphi}_n$ are mutually orthogonal.

Since solutions of a system of equations (2) and (16) are found with respect to the frequency (with the constant-value bias field), or with respect to the bias field (with the constant-value frequency), one should have a *mutual matching* between the frequency (see figure 3) and the

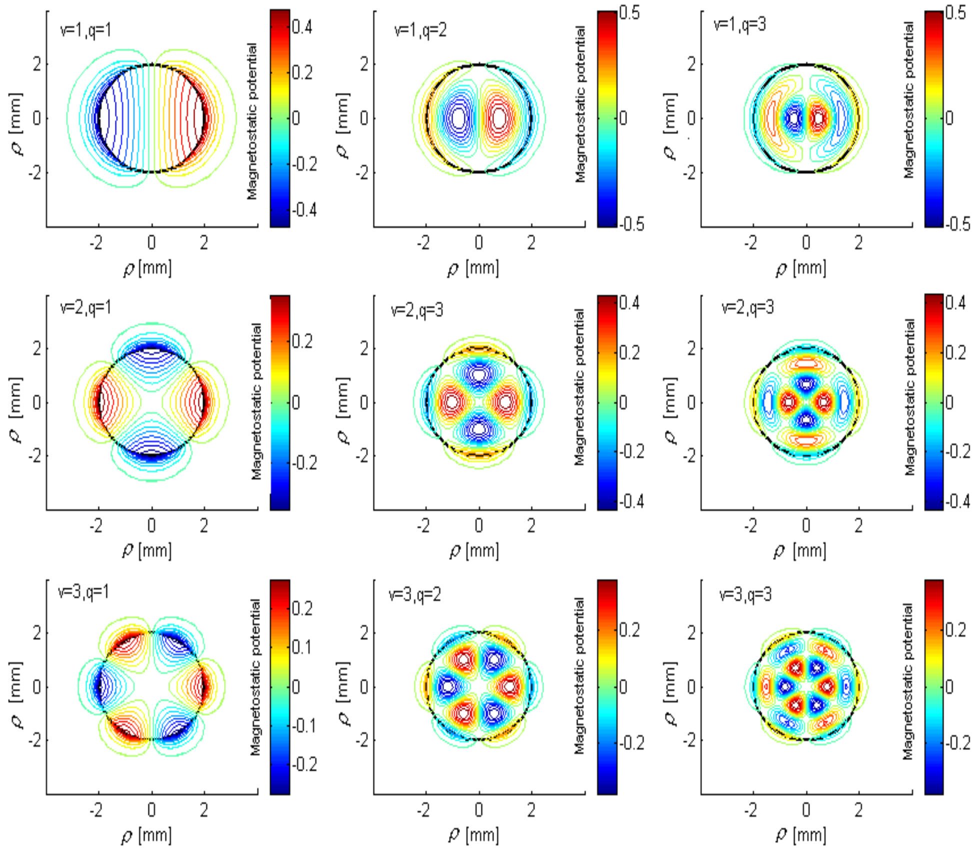

Figure 7. MS-potential distributions with respect to an azimuth coordinate.

magnetic-field (see figure 4) spectra. This matching is illustrated in figure 10 for the first three peaks ($q = 1, 2, 3$) of the case of $\nu = 1$. In figures 10(a), (b), and (c) we see the peak positions for the frequency spectrums, respectively, at $H_0|_{q=1,\nu=1}$, $H_0|_{q=2,\nu=1}$, and $H_0|_{q=3,\nu=1}$ (see figure 4). In fact, we sequentially replace every peak from the frequency spectrum at the same frequency f' by a sequence of the DC magnetic field values.

For the situation shown in figure 10 (when we sequentially replace every peak from the frequency spectrum *at the same frequency* ($\omega_m = \omega_n = \omega'$) by a sequence of the DC magnetic field values), the functions $\tilde{\varphi}_m$ and $\tilde{\varphi}_n$ are mutually orthogonal. This statement is evident. In fact, in accordance with the above consideration, any 'in-plane' eigenfunctions are mutually orthogonal for the monochromatic process. This means that for the magnetic-field (see figure 4) spectrum of a ferrite disc, the ψ function can be expanded by complete-set 'flat' functions $\tilde{\varphi}$. So for the frequency spectrum the eigenfunctions are also mutually orthogonal.

5. The case of $a^{(2)} \neq 0$

In the above consideration we showed an analysis of the steady-state functions. To analyse *transitions* between the steady-state energy levels one has to solve equation (4) in the time and space domains. This very interesting problem should be a subject for future investigation.

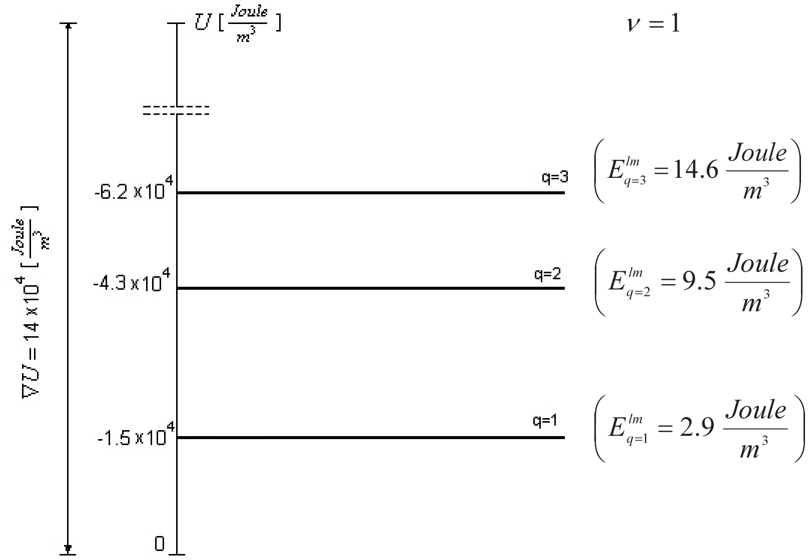

Figure 8. First three levels of potential energy ($\nu = 1$).

Nevertheless, there are some aspects of the transitional regimes one can discuss within the framework of this paper.

The average energy of a ferrite disc can be expressed by equation (13) only for a given quantity of a bias magnetic field. With variation of the quantity of the bias magnetic field, one has variation of the 'potential energy' of a ferrite sample. In this case, we should take $a^{(2)} \neq 0$ in equation (4). With a clear similarity to the Schrödinger equation, one can see that coefficient $a^{(2)}(z)$ in equation (4) corresponds to the potential-energy function. So, considering equation (4) as an operator equation with respect to a wavefunction ψ, one can conclude that the first term in the left-hand side of equation (4) describes an operator of kinetic energy, while the second term describes the potential energy operator. Since the coefficient $a_n^{(2)}$ in equation (5)—the potential energy operator—is dependent on neither frequency ω nor wavenumber β_n, one has two ways to define coefficient $a_n^{(1)}$: by means of taking derivatives over (β_n^2) in equation (5), or by means of taking derivatives over ω in equation (5). As a result, one can write:

$$a_n^{(1)} = -\mathrm{i}\frac{1}{\mathrm{d}(\beta_n^2)/\mathrm{d}\omega} = -\mathrm{i}\frac{1}{2}\frac{\mathrm{d}^2\omega}{\mathrm{d}\beta_n^2}. \tag{22}$$

The above equality is valid when one assumes that β_n^2 is proportional to ω. The behaviour of quasiparticles characterized by such a coefficient $a^{(1)}$ differs from the behaviour of the light magnons. Such quasiparticles we will conventionally call the '*quasimagnons*' (qm). One obtains the following expression for the normalized average MS energy of 'flat mode' q, in the case of 'quasimagnons':

$$E_q^{(\mathrm{qm})} = -\frac{1}{2a_q^{(1)}}\mathrm{i}\omega\mu_0 g = \frac{1}{2}\omega\mu_0 g\frac{\mathrm{d}\left(\beta_q^{(\mathrm{F})}\right)^2}{\mathrm{d}\omega}. \tag{23}$$

Based on the definition of the effective mass for quasiparticles in a crystal [21], one can introduce the notion of an effective mass of a 'quasimagnon'. For MS mode q the effective

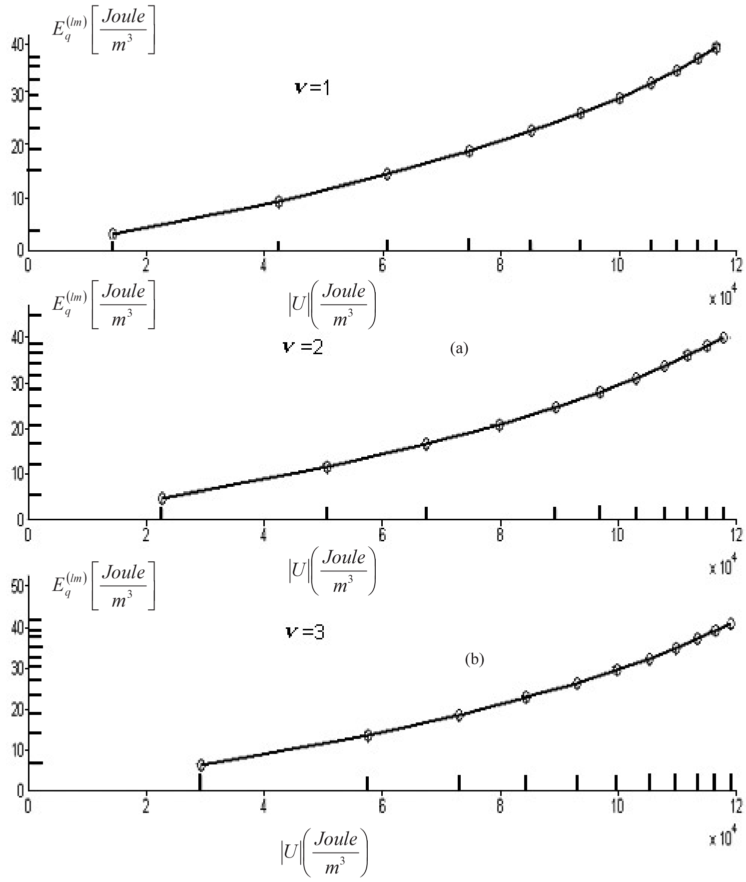

Figure 9. Appropriateness between the $E_q^{(\mathrm{lm})}$ and $|U_q|$ levels: (a) for dipole-type light magnons ($\nu = 1$), (b) for quadrupole-type light magnons ($\nu = 2$), for hexapole-type light magnons ($\nu = 3$).

mass of a 'quasimagnon' is expressed as:

$$\frac{1}{\left(m_{\mathrm{eff}}^{(\mathrm{qm})}\right)_q} = \frac{1}{\hbar}\frac{\mathrm{d}^2\omega}{\mathrm{d}\left(\beta_q^{(\mathrm{F})}\right)^2}. \tag{24}$$

In accordance with the dispersion properties of MS waves in an axially magnetized ferrite rod [15], one can see that an effective mass of a 'quasimagnon' is *negative*. However, an average energy $E_q^{(\mathrm{qm})}$ expressed by equation (23) should be a positive quantity. This stipulates the conclusion that to describe the negative-mass 'quasimagnons' one should use a notion of negative frequency ω. The 'negativeness' of frequency ω is clearly demonstrated by figure 10. One can see that a spectrum 'moves' in a negative direction on the frequency axis as we pass from the 'top value' of the bias magnetic field.

It is possible to show that the levels calculated based on equation (23) do not give a regular spectral picture. The eigenfunctions corresponding to the energy eigenstates defined by expression (23) are not mutually orthogonal. The transitional functions considered above demonstrate some interesting physical aspects, but the real analysis, as we discussed above, should be based on the time- and space-domain calculations.

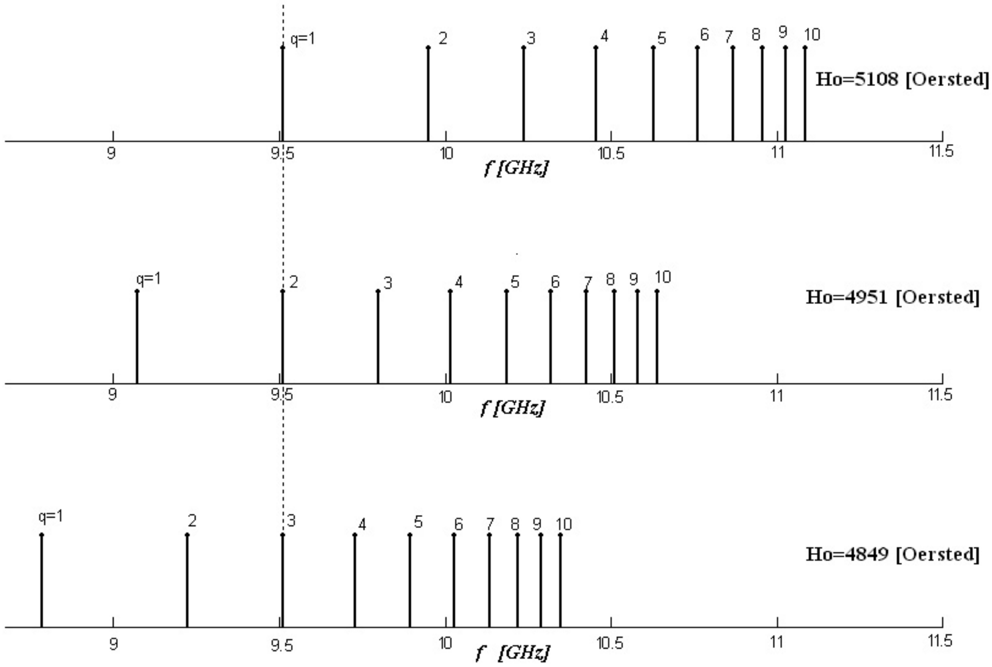

Figure 10. Mutual matching between the frequency and the magnetic-field spectra.

6. On the quantization of magnetization for MS oscillations

We have shown that in a ferrite disc resonator every MS-wave oscillating mode is characterized by a certain energy level. Now we come to the question: what is the correlation between the energies of the MS oscillations analysed above and the density of magnetization energy?

Quantization of the magnetization *of a whole ferrite-particle system* may take place if there is *a spectral problem for a magnetization field* or if a magnetization field can be expressed by the compete-set scalar functions. Both these cases do not occur for MS oscillations in a normally magnetized ferrite disc. For the biasing magnetic field directed along the z-axis, the RF magnetization in the MS description is written in cylindrical coordinates as [17]:

$$\begin{aligned} m_\rho &= -\chi\frac{\partial\psi}{\partial\rho} - \mathrm{i}\chi_\alpha\frac{1}{\rho}\frac{\partial\psi}{\partial\alpha}, \\ m_\alpha &= \mathrm{i}\chi_\alpha\frac{\partial\psi}{\partial\rho} - \chi\frac{1}{\rho}\frac{\partial\psi}{\partial\alpha}, \\ m_z &= 0, \end{aligned} \tag{25}$$

where χ and χ_a are, respectively, diagonal and off-diagonal components of the susceptibility tensor $\overleftrightarrow{\chi}$. For a dominant 'thickness' mode described by the known function $\tilde{\xi}(z)$ and q 'flat' mode described by $\tilde{\varphi}_q(\rho,\alpha)$ (see equation (1)), one has:

$$\begin{aligned} (m_\rho)_q &= A_q\tilde{\xi}\left(-\chi\frac{\partial\tilde{\varphi}_q}{\partial\rho} - \mathrm{i}\chi_\alpha\frac{1}{\rho}\frac{\partial\tilde{\varphi}_q}{\partial\alpha}\right), \\ (m_\alpha)_q &= A_q\tilde{\xi}\left(\mathrm{i}\chi_\alpha\frac{\partial\tilde{\varphi}_q}{\partial\rho} - \chi\frac{1}{\rho}\frac{\partial\tilde{\varphi}_q}{\partial\alpha}\right), \\ (m_z)_q &= 0. \end{aligned} \tag{26}$$

Based on these expressions, one can introduce the corresponding 'in-plane' differential operators:

$$\hat{m}_\rho = -\chi \frac{\partial}{\partial \rho} - \mathrm{i}\chi_\alpha \frac{1}{\rho}\frac{\partial}{\partial \alpha}, \qquad \hat{m}_\alpha = \mathrm{i}\chi_\alpha \frac{\partial}{\partial \rho} - \chi \frac{1}{\rho}\frac{\partial}{\partial \alpha}. \tag{27}$$

Now we define the operator:

$$\hat{m}_\perp^2 = \hat{m}_\rho^2 + \hat{m}_\alpha^2. \tag{28}$$

This operator is expressed as:

$$\hat{m}_\perp^2 = (\chi^2 - \chi_a^2)\left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2}{\partial \alpha^2}\right) = \gamma M_0 \left(\nabla_\perp^2 - \frac{1}{\rho}\frac{\partial}{\partial \rho}\right). \tag{29}$$

Let us suppose, as a particular case, that a lateral cylindrical surface of a ferrite disc is a perfect magnetic wall. In this case one has the same domains of definition of operators $\hat{m}_\perp^2$ and $\nabla_\perp^2$. Unlike energy operator $\hat{F}_\perp$ [7], operator $\hat{m}_\perp^2$ is not proportional to the Laplace 'in-plane' operator $\nabla_\perp^2$. Evidently, operators $\hat{F}_\perp$ and $\hat{m}_\perp^2$ *do not commute with each other*. This means that for every oscillating MS 'flat' mode q (with a dominant 'thickness' mode), energy E_q described by the MS-potential function (see equation (13)) is not equal to the oscillating energy of the RF magnetization in a ferrite sample. This also means that for every energy eigenstate E_q, the value $m_q^2 = (m_\rho)_q^2 + (m_\alpha)_q^2$ is not a stationary-state quantity. One can suppose that in accordance with equation (26), for every (normalized) MS-potential mode there is a 'magnetization mode'. But these 'magnetization modes' are not orthonormalized ones. In other words, the energy of the magnetization does not belong to the orthonormal space of energy. The fact that in our case we have quantization of MS energy (energy eigenstates of MS oscillations) does not stipulate quantization of the magnetization energy in MS-wave oscillations.

The above statement that one does not have quantization of the magnetization energy in MS-wave oscillations can also be illustrated in another way. Let us suppose that operator $\hat{m}_\perp^2$ has eigenvalues, which we denote as m^2, and 'flat' scalar eigenfunctions $\tilde{\varsigma}$ different from function $\tilde{\varphi}$. In this case, we have the equation:

$$\hat{m}_\perp^2 \tilde{\varsigma} = m^2 \tilde{\varsigma} \tag{30}$$

or

$$\frac{\partial^2 \tilde{\varsigma}}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 \tilde{\varsigma}}{\partial \alpha^2} - \frac{m^2}{\chi^2 - \chi_a^2}\tilde{\varsigma} = 0. \tag{31}$$

Here $m^2 = m_\rho^2 + m_\alpha^2$ is the quantity defining the density of the magnetization energy [17]. Because of cylindrical symmetry and the supposition that functions $\tilde{\varsigma}$ and $\tilde{\varphi}$ might be at the same azimuth symmetry, we have $\tilde{\varsigma} \sim \mathrm{e}^{-\mathrm{i}\nu\alpha}$. Equation (30) is written as:

$$\frac{\partial^2 \tilde{\varsigma}}{\partial \rho^2} - \frac{\nu^2}{\rho^2}\tilde{\varsigma} + Q^2 \tilde{\varsigma} = 0, \tag{32}$$

where we have denoted:

$$Q^2 = \frac{m^2}{\chi_a^2 - \chi^2}. \tag{33}$$

Now let us introduce a new real quantity n, which is defined by the following equation:

$$\frac{4n^2 - 1}{4} = \nu^2. \tag{34}$$

Based on equation (34) we rewrite equation (32) as:

$$\frac{\partial^2 \tilde{\varsigma}}{\partial \rho^2} + \left(Q^2 - \frac{4n^2 - 1}{4\rho^2} \right) \tilde{\varsigma} = 0. \quad (35)$$

One has the following solution for function $\tilde{\varsigma}$ [22]:

$$\tilde{\varsigma} = \text{constant } \sqrt{\rho} J_n(Q\rho) e^{-i\nu\alpha} = \text{constant } \sqrt{\rho} J_{\pm\frac{1}{2}\sqrt{4\nu^2+1}}(Q\rho) e^{-i\nu\alpha}. \quad (36)$$

For the essential boundary conditions, functions $\tilde{\varphi}$, being expressed by Bessel's functions, are mutually orthogonal. There is no formal evidence of orthogonality of functions $\tilde{\varsigma}$. Moreover, if one supposes that functions $\tilde{\varphi}$ form a complete set of scalar functions and that operators $\hat{m}_\perp^2$ and $\nabla_\perp^2$ have the same domains of definition, another type of scalar function $\tilde{\varsigma}$ cannot be the complete-set function.

7. The MS-mode spectral problem in a ferrite disc with non-homogeneous DC magnetic field

It was supposed in our analysis that the internal DC magnetic field is homogeneous. In this case one can really formulate the spectral problem. At the same time, because of the demagnetizing effects, the internal DC magnetic field in a ferrite disc is essentially non-homogeneous. This should have a strong affect on the spectral picture. An analysis of the spectral peak positions for MS oscillations, taking into account the DC magnetic field non-homogeneity, was made in [4]. Based on the analysis in [4] one cannot, however, determine the character of eigenfunctions. Therefore the 'spectral portrait' of MS oscillations in discs with non-homogeneous internal DC magnetic field becomes unclear. So *the physics of interaction of such a particle with the external RF fields becomes absolutely unclear*.

The absorption peaks are interpreted in [4] to be caused by magnetostatic waves propagating radially across the disc with the DC-field dependent wavenumber in a plane of a YIG film. The mode numbers are determined based on the well-known Bohr–Sommerfeld quantization rule. In the definition of the spectral peak positions, the Yukawa and Abe model gives good agreement with experiment. This fact is illustrated in figure 11 for MS modes excited by the homogeneous RF magnetic field (here we use the Yukawa and Abe notation for the odd mode numbers, so the Yukawa and Abe mode enumeration $n = 1, 3, 5, \ldots$ corresponds to our dipole-type mode enumeration $q = 1, 2, 3, \ldots$ for $\nu = 1$). In spite of such good agreement, one cannot, however, rely on the Yukawa and Abe model to physically describe the experimental situation of interaction of small ferrite discs with the external RF fields. When being excited by the homogeneous RF magnetic field, a small ferrite disc should be considered as a magnetic dipole with evident *azimuth variations of the 'in-plane' MS-potential function*. At the same time, in the Yukawa and Abe model the 'in-plane' MS-potential-function distribution is supposed to be azimuthally non-dependent.

For a normally magnetized thin-film ferromagnetic disc having small thickness-to-diameter ratio, the demagnetizing field can be considered just as the radius-dependent function. The internal DC magnetic field is determined in [4] as:

$$H_i(\rho) = H_0 - H_a - 4\pi M_0 I(\rho), \quad (37)$$

where H_0 and H_a are the applied and the anisotropy fields, respectively, and $I(\rho)$ is the demagnetizing factor. In this case, however, the standard cylindrical-symmetry problem for MS modes cannot be solved. It is not difficult to show that when the permeability-tensor components are dependent on a radial coordinate, the differential equation for the MS-potential functions:

$$\nabla \cdot (\overleftrightarrow{\mu}(\rho) \cdot \nabla \psi) = 0 \quad (38)$$

does not have separation of variables in a cylindrical coordinate system.

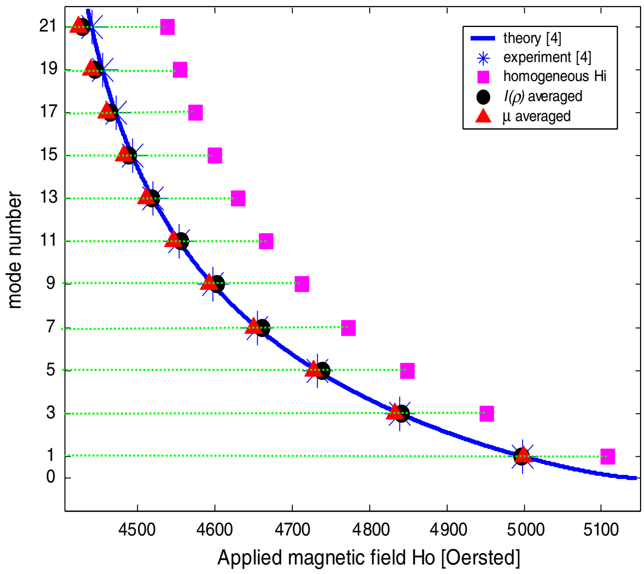

Figure 11. Mode numbers with respect to applied magnetic field for different calculation methods.

The above spectral properties of MS modes in a ferrite disc were analysed based on the disc data given in [4], but under the assumption of a homogeneous internal DC magnetic field and without taking into account the anisotropy field. This means that the internal field was calculated based on equation (37), but with $H_a = 0$ and $I(\rho) = 1$. In this case, however, the spectral peak positions are rather far from the experimental peak positions. This is illustrated in figure 11 by squares.

To take into account the internal field inhomogeneity preserving, at the same time, the quantized 'spectral portrait' of MS oscillations in a ferrite disc, one has to develop certain models of averaging. An evident criterion of acceptability of an averaging model should be the fact that the total spectral picture (but not the separated modes) will be shifted to the corresponding experimental-spectrum picture. In [23], two averaging models were developed: one based on radial averaging of the function $\overset{\leftrightarrow}{\mu}(\rho)$ and another, the averaging of the function $I(\rho)$. The calculation results depicted in figure 11, by circles for $I(\rho)_{\text{average}}$ and by triangles for $\overset{\leftrightarrow}{\mu}(\rho)_{\text{average}}$, give good agreement with experimental peak positions.

8. Conclusion

Quantum effects are rarely observed through macroscopic measurements because statistical averaging over many states usually masks all evidence of discreteness. One of the notable exceptions is a quantum-mechanical effect in the magnetization of a macroscopic ferrite sample: quantum coherence of magnetic-dipolar oscillations in thin-film ferrite discs. In quasi-two-dimensional systems, the dipolar interaction can play an essential role in determining the magnetic properties. In these systems the short-range exchange interactions alone are not necessarily sufficient to establish a ferromagnetically ordered ground state. The theory of the magnetic-dipolar mode spectra is based on the notion of magnetostatic-potential wavefunctions. In the case of magnetic-dipolar oscillations in a normally magnetized thin-film ferrite disc, magnetostatic-potential wavefunctions acquire a special physical meaning.

Ferromagnetic resonators with MS oscillations can be considered in microwaves as point (with respect to the external electromagnetic fields) particles. MS oscillations in a small ferrite disc resonator can be characterized by a *discrete spectrum of energy levels*. This fact allows the analysis of the MS oscillations in a similar way to quantum mechanical problems. Such a macroscopically quantum picture underlies the physics of *magnetoelectric* discrete spectrums recently observed in ferrite discs with surface electrodes [13]. A special interest in the energy spectra of such a small structural element—artificial magnetic atoms—may be found in the fields of microwave artificial composite materials, microwave spectroscopy, and, probably, quantum computation [24].

From the above analysis it follows that the energy quantization (described by the MS potential properties) can be regarded as a collective effect of quasiparticles—the light magnons. In other words, the MS-wave phenomena in a special *macrodomain*—a ferrite disc resonator—can be simply reduced to the case of a many-particle correlated system. The (real, 'heavy') magnon mass in YIG is approximately six times larger than the free electron mass [17]. At the same time, our estimations show that the light magnon effective mass $m_{\mathrm{eff}}^{(\mathrm{lm})}$ (for YIG disc resonators with parameters corresponding to the data of experiments [4]) is a very small quantity, which is about 10^8 times less than the free electron mass. This fact is very clear since MS oscillations take up an 'intermediate position' between the electromagnetic and exchange oscillations. On the one hand, to describe these oscillations we can put the phenomenological exchange constant equal to zero. On the other hand, the MS oscillations are described in neglect of the electric displacement current in the Maxwell equations. So the quantity of the light-magnon effective mass should be between the 'heavy'-magnon effective mass (very big) and zero mass of the photon.

As we have discussed in [7, 11] and analysed in detail in the present paper, because of the possibility of formulating the energy eigenvalue problem for a normally magnetized flat ferrite disc, one obtains a complete *discrete spectrum of energy levels for a whole ferrite-particle system*. In such a case MS potential functions can be considered as the *probability distribution functions*. The Schrödinger-like equation written for the MS-potential wavefunction shows that in a ferrite disc resonator MS modes can diagonalize the total magnetic energy. One of the main features of the problem is the fact that there are two types of spectra: (a) the spectra obtained for the constant-value bias magnetic field and (b) the spectra obtained for the constant-value frequency.

Our analysis is based on an assumption that the internal DC magnetic field is homogeneous. In this case one can really formulate the spectral problem. At the same time, because of the demagnetizing effects, the internal DC magnetic field in a ferrite disc is essentially non-homogeneous. This should strongly affect the spectral picture. To take into account the internal field non-homogeneity preserving, at the same time, the quantized 'spectral portrait' of MS oscillations in a ferrite disc one has to develop certain models of averaging. The suggested models of averaging give good agreement with experimental peak positions. Based on the above models of averaging, the energy spectra in MS-wave ferrite discs taking into account inhomogeneity of the internal DC magnetic field can be effectively calculated. This makes solvable the problem of interaction of such ferrite particles with the external RF fields.

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